Effect of non adiabatic switching of dynamic perturbations in 1d Fermi systems

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Abstract

We study a two-dimensional fermionic QFT used to model 1D strongly correlated electrons in the presence of a time-dependent impurity that drives the system out of equilibrium. In contrast to previous investigations, we consider a dynamic barrier switched on at a finite time. We compute the total energy density (TED) of the system and establish two well defined regimes in terms of the relationship between the frequency of the time-dependent perturbation Ω and the electron energy ω. Finally, we derive a relaxation time t_R such that for times shorter than t_R the finite-time switching process is relevant.

Keywords: field theory, non equilibrium, bosonization, Luttinger.
Pacs: 71.10.Pm, 05.30.Fk

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Low dimensional field theories are objects of sustained interest. In particular they are useful to describe the behavior of strongly anisotropic physical systems such as organic conductors [1], charge transfer salts [2], quantum wires [3], and carbon nanotubes [4]. In this condensed matter context, one of the most widely studied 1D theory is the so called "g-ology" model [5], which displays the Luttinger liquid behavior characterized by spin-charge separation and interaction-dependent power-law correlation functions [6]. Many of the advances in the understanding of this model have been accomplished through the use of the renormalization group and the bosonization procedure, and in the context of equilibrium situations, i.e. in the absence of time-dependent interactions. However, as it is well known, in nature, thermal equilibrium is more an exception than a rule. In fact, in recent years there has been an increasing activity focused on the study of out of equilibrium low-dimensional electronic systems [7]. Thus, it becomes natural to consider the formulation of field-theoretical methods that can be applied to this kind of systems. To this aim, we have recently shown how to combine the Closed Time Path (CTP) method of Schwinger and Keldysh [8] with a path-integral approach to bosonization [9] in order to analyze the g-ology model in the presence of a time-dependent dynamic barrier [10]. On the other hand, as far as we know, all previous works on 1D electronic systems with time-dependent impurities (including ref.[10]) use a large-time approximation in order to treat the correlation functions. In this case the effect of a non adiabatic switching of the interaction can be disregarded and the total energy density (TED) becomes an harmonic function of time which temporal average can be easily defined and computed. An obvious and drastic consequence is that no transient process can be predicted. The main purpose of this work is to reconsider this problem by taking into account the role of a sudden, finite-time switching process. This will enable us to show how the TED evolves from short to long-time regimes.

We are interested in computing the TED for a Luttinger system defined by

$$S = S_0 + S_{int} + S_I, \tag{1}$$

where S_0 is the unperturbed action (in the condensed matter context it is thought of as a linearized free dispersion relation):

$$S_0 = \int_C d^2x \, \bar{\Psi} i\partial\!\!\!/ \Psi, \tag{2}$$

where $\int_C$ indicates that the integration is defined along the time contour usually defined in the CTP formalism [8]. S_{int} describes the forward scattering of spinless fermions (electrons):

$$S_{int} = -\frac{1}{2} \int_C d^2x \, d^2y \, (\bar{\Psi}\gamma_\mu\Psi)(\mathbf{x}) \; V_{(\mu)}(\mathbf{x}-\mathbf{y}) \; (\bar{\Psi}\gamma_\mu\Psi)(\mathbf{y}) \tag{3}$$

where $\mathbf{x} = (x,t)$ and the fermionic currents $\bar{\Psi}\gamma_\mu\Psi$ are coupled through distance-dependent functions, $V_{(\mu)}(\mathbf{x}-\mathbf{y})$. In terms of these potentials one can make direct contact with

the forward-scattering sector of the "g-ology" model currently used to describe different scattering processes characterized by coupling functions g_1, g_2, g_3 and g_4 [5]. Neglecting processes associated to large momentum transfers, only the forward scattering couplings g_2 and g_4 play a role. The relation between these strengths and our potentials are given (in Fourier space,$\mathbf{p} = (p, \omega)$) by

$$\begin{aligned} g_2(\mathbf{p}) &= \tfrac{1}{2}(V_{(1)}(\mathbf{p}) + V_{(0)}(\mathbf{p})) \\ g_4(\mathbf{p}) &= \tfrac{1}{2}(V_{(0)}(\mathbf{p}) - V_{(1)}(\mathbf{p})). \end{aligned} \tag{4}$$

From now on we shall disregard the momentum dependence of the potentials, i.e. only short-range interaction will be taken into account. The action S_I describes the interaction between the electrons and a localized time-dependent perturbation:

$$S_I = \int_C d^2x \, \bar{\Psi} \, \gamma_0 \, \Delta(x,t) \, \Psi, \tag{5}$$

where γ_0 is a Dirac matrix and $\Delta(x,t)$ is a function that contains the details of the perturbation, i.e. the way in which the interaction is switched on in time and the form in which it is localized in space. Please note that, for simplicity, only forward scattering between the electron and the impurity is considered. As we shall see later, this is not a bad approximation if weak couplings and short times are taken into account. Besides, in some experimental arrangements one can reasonably argue that backscattering effects will not be important (see, for instance [11]). To be specific we choose a particular form for the perturbation:

$$\Delta(x,t) = \lambda \, \sin(\Omega t) \, \Theta(t)[\Theta(x + a/2) - \Theta(x - a/2)] \tag{6}$$

where $\Theta(t)$ is the Heaviside function. We then have a separable harmonic perturbation suddenly switched on, consisting of a barrier of width a and height λ that oscillates in time with frequency Ω. As we shall see, this choice allows us to make contact with previous works on the effect of time-dependent perturbations on one-dimensional systems. Indeed, a similar potential (with an additional static term) was considered by Büttiker and Landauer in their study of the traversal time for tunneling [12]. More recently, and in the context of Luttinger liquids, temporal harmonic perturbations were analyzed for both forward [13] and backscattering [14] impurities. Let us stress that neither of these authors includes the effect of switching, which is our main motivation.

Let us now focus our attention on the TED, which in the Wigner representation can be expressed in terms of the correlation function as

$$n(\omega, x, t) = -i \int_{-\infty}^{\infty} d\tau \exp(i\omega\tau) \, G_{-+}(x, x, t + \tau/2, t - \tau/2), \tag{7}$$

where the right and left fermionic propagators are time-ordered along a time contour C:

$$G_C^{R,L} = \begin{pmatrix} G_{++}^{R,L} & G_{+-}^{R,L} \\ G_{-+}^{R,L} & G_{--}^{R,L} \end{pmatrix}. \tag{8}$$

The subscripts + and − refer to fields defined in the upper and lower branches of C, respectively, corresponding to forward (+) and backward (−) time evolution. From now on we shall restrict our analysis to the TED corresponding to right-movers. Similar results can be obtained for left-moving particles.

Using the technique described in [10] we can factorize the Green function as

$$G_C(x,y,t,t') = G_C^{\gamma}(x-y,t-t')\exp\big[\beta(x,t)-\beta(y,t')\big], \tag{9}$$

where the function $G_C^{\gamma}(x-y,t-t')$ is the equilibrium propagator for a Luttinger liquid with its characteristic interaction dependent exponent $\gamma = (1/4)(K+K^{-1}-2)$, ($K = \sqrt{\frac{1+g_4/\pi-g_2/\pi}{1+g_4/\pi+g_2/\pi}}$) and

$$\beta(x,t) = \frac{-i\lambda}{2\Omega}\Theta(t)\{e^{i\Omega t}F(x,a,\Omega) + e^{-i\Omega t}F(x,a,-\Omega)\} \tag{10}$$

with

$$\begin{aligned} F(x,a,\Omega) \;&= \Theta(-(x+a/2))\exp[-i\Omega(x+a/2)] - \Theta(-(x-a/2))\exp[-i\Omega(x-a/2)] + \\ &\quad +\Theta(x+a/2) - \Theta(x-a/2). \end{aligned} \tag{11}$$

Since we are mainly concerned with the effect of the switching process, we start by studying the free case in the presence of the time-dependent impurity ($\gamma = 0$). For this case the correlation function is given by

$$G_{-+}^{(0)}(x,t+\tau/2;x,t-\tau/2) = \frac{i}{2\pi}\frac{1}{\alpha_0+i\tau}. \tag{12}$$

Expanding the exponential in eq. (9), expressing the β-dependent factor in terms of the binomial expansion and replacing (9) in (7), after some algebra one can write the TED

as

$$n(\omega,x,t)_{\gamma=0}=\Theta(\omega)+\sum_{n=1}^{\infty}\Big(\frac{-i\lambda}{2\Omega}\Big)^n\sum_{j=0}^{n}\frac{1}{(n-j)!j!}\big(F(x,a,\Omega)\big)^{n-j}\big(F(x,a,-\Omega)\big)^{j}\exp\{i\Omega t(n-2j)\}\times$$
$$\times\Big[\Theta(\omega+(n-2j)\Omega/2)+\frac{i}{2\pi}\big(\mathrm{Ci}[(2t+i\alpha_0)(\omega+(n-2j)\Omega/2)]-i\,\mathrm{Si}[(2t+i\alpha_0)(\omega+(n-2j)\Omega/2)]+$$
$$+(-1)^{n+1}\big(\mathrm{Ci}[(2t-i\alpha_0)(-\omega+(n-2j)\Omega/2)]-i\,\mathrm{Si}[(2t-i\alpha_0)(-\omega+(n-2j)\Omega/2)]\big)\big)\Big]+$$
$$+\sum_{n=2}^{\infty}\Big(\frac{-i\lambda}{2\Omega}\Big)^n\sum_{j=1}^{n-1}(-1)^j\sum_{k=0}^{j}\sum_{l=0}^{n-j}\frac{1}{(j-k)!i!(n-j-l)!l!}\big(F(x,a,\Omega)\big)^{n-l-k}\big(F(x,a,-\Omega)\big)^{l+k}\times$$
$$\times\frac{i\Theta(t)}{2\pi}\exp\{i\Omega t(n-2k-2l)\}\Big[\mathrm{Ci}\big[(2t+i\alpha_0)(\omega+(n-2j-2l+2k)\Omega/2)\big]+$$
$$-i\,\mathrm{Si}\big[(2t+i\alpha_0)(\omega+(n-2j-2l+2k)\Omega/2)\big]-\mathrm{Ci}\big[(-2t+i\alpha_0)(\omega+(n-2j-2l+2k)\Omega/2)\big]+$$
$$+i\,\mathrm{Si}\big[(-2t+i\alpha_0)(\omega+(n-2j-2l+2k)\Omega/2)\big]\Big]. \quad (13)$$

Let us mention that the first term in the above formula is the equilibrium TED. Please observe that for $t\to-\infty$, i.e. for long times in the past, only this first term remains and the TED recovers its equilibrium value: $n(\omega,x,t)_{\gamma=0}=n_0(\omega)=\Theta(\omega)$, as expected. We stress that the inclusion of the factor $\Theta(t)$ in the perturbation gives rise to the appearance of non harmonic contributions to the TED. These contributions manifest through the functions cosine-integral (Ci) and sine-integral (Si). For large positive times only harmonic terms survive. In this regime the TED is a harmonic superposition of equilibrium TED's centered in integer multiples of $\Omega/2$, with coefficients that depend on the geometry and strength of the time-dependent interaction. Only in this regime a time average (over the period of the interaction) does not depend on the temporal interval and leads to an averaged TED which is a superposition of equilibrium TED's centered in integer multiples of Ω [13] [10]. Some of these features can be globally seen in figures 1 and 2, where we show the behavior of $n(\omega,x,t)_{\gamma=0}$ as function of ω and t, for fixed x inside the barrier ($-a/2<x<a/2$). For comparison purposes, in figure 1 we disregarded the effect of the non adiabatic switching, whereas in figure 2 the $\Theta(t)$ function has been included. We have set $\lambda=2$, $\Omega=1$ and $a/2-x=1$.

When one focus the attention on the behavior of the TED as the energy ω is varied, another interesting feature is revealed. We can establish two well defined regimes in terms of the relationship between the external frequency Ω and the electron energy ω. Indeed, for $\Omega/\omega\ll1$ and positive times, the electrons "do not see" the harmonic perturbation and the corresponding TED is equal to $n_0(\omega)$. So, this is the large energy region where the effect of the impurity is negligible. At this point we should emphasize that the impurity considered in this paper does not include a static term. Had we considered such a contribution we would have found that the large energy regime is dominated

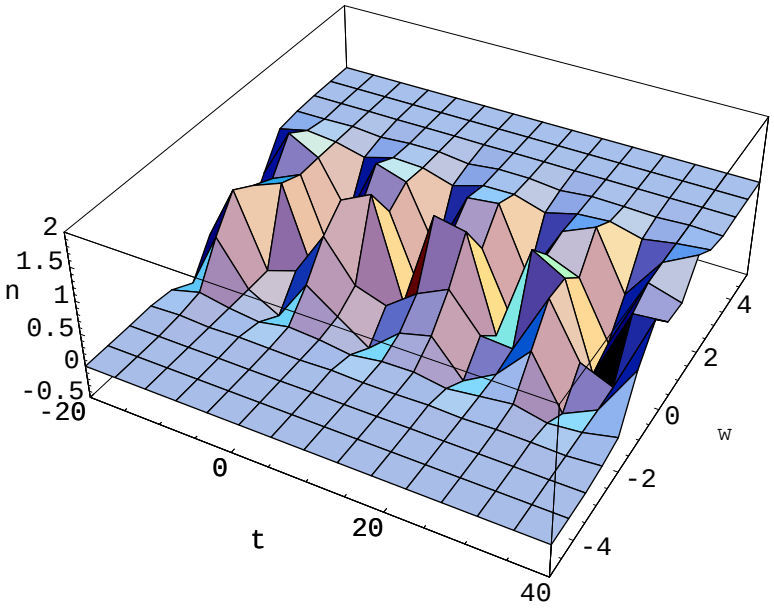

Figure 1: : $n(\omega, t)_{\gamma=0}$ as function of ω and t, without finite-time non adiabatic switching.

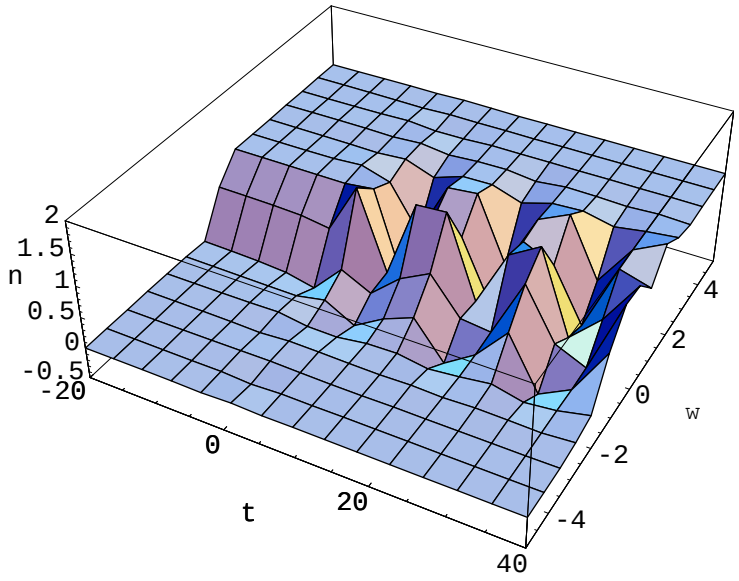

Figure 2: : $n(\omega, t)_{\gamma=0}$ as function of ω and t, with finite-time non adiabatic switching.

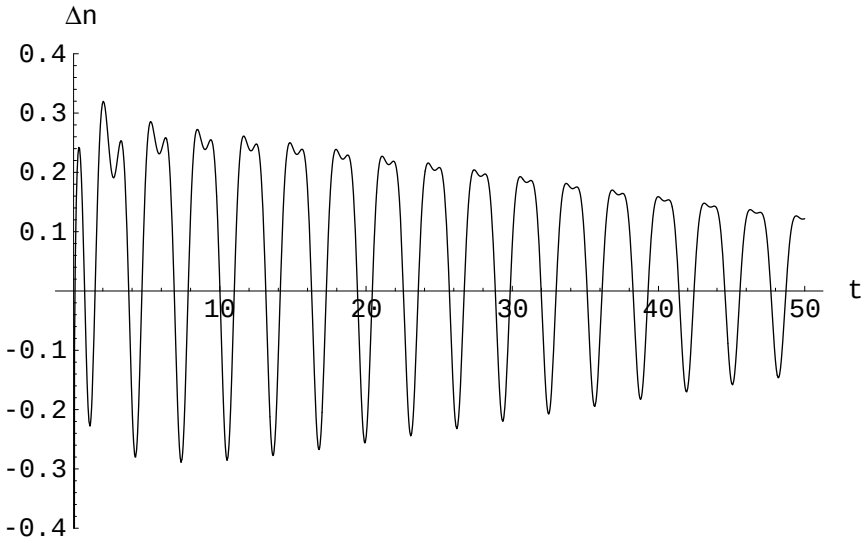

Figure 3: : $\Delta n(t)$ in the low energy regime, for $\gamma = 0$, $\Omega = 1$, $\omega = 0.01$.

by the static impurity. The transient effects arising in this case have been extensively investigated in connection to the X-ray problem and the orthogonality catastrophe, for both Fermi [15] and Luttinger liquids [16]. In our framework, however,in order to make a precise description of the X-ray problem, one should modify the temporal behavior of the impurity, i.e. the function associated to the switching process, such that a switching off time is also incorporated.

Going back to the analysis of our TED, one sees that for $\Omega/\omega \gg 1$ it is greatly affected by the impurity. In the absence of switching (figure 1) the electrons gain or lose energy quanta of value $\pm n\Omega$ (this is clearly seen when one considers the time average). The same occurs when switching is taken into account (figure 2), for sufficiently large times. Of course, there is also a crossover region given by $\Omega/\omega \cong 1$. At this point we can make contact with previous studies on the traversal time for tunneling in the context of the quantum mechanics of a particle in the presence of a dynamic barrier[12]. From the uncertainty principle we know that the traversal time τ satisfies $\tau \sim 1/\omega$, which leads to $\tau\Omega \cong 1$, a result which is consistent with the findings of ref.[12].

In order to perform a quantitative description of the transient process which is now accesible due to the introduction of a finite-time switching mechanism, we found useful to consider the difference between TED's with and without the inclusion of the temporal Heaviside function. We call this difference $\Delta n(t)$. In Figure 3 we plot $\Delta n(t)$ in the low energy regime ($\Omega = 1$,$\omega = 0.01$). From this figure one can see that it is possible to define and compute a relaxation time t_R such that for times larger than t_R the TED coincides with the one obtained when the impurity is always switched on. Thus, $t < t_R$

defines a temporal region in which the switching process is relevant. We have analyzed the envelope of $\Delta n(t)$ for many different values of ω, always in the low energy region $\Omega/\omega \gg 1$. In other words, we fit the envelope with an exponential decay $b \exp(-t/t_R)$. This procedure allowed us to find $t_R = 1/(2\omega)$. The variation of λ and Ω only affects the values of b.

Let us now say a few words on the role of backscattering from the impurity. If we add to the Lagrangian the corresponding term, of the form $\bar{\Psi}\,\Delta_{back}(x,t)\,\Psi$, of course, the problem of determining the Green's functions is not exactly solvable any more and one is forced to make a perturbative calculation. For a general interaction of the form $\Delta_{back}(x,t) = \lambda_{back}\, f(x,t)$, where the new coupling constant λ_{back} is assumed to be of the same order of λ, it is easy to show that up to first order in these perturbative parameters, only the forward scattering term affects the behavior of the TED. This means that in the short-time regime ($t < t_R$) and for weak coupling, only forward scattering from the impurity contributes to the TED. In the large-time regime, the first order contribution in λ gives a zero average and then one is led to a higher order calculation in which backscattering effects would eventually contribute in a non trivial way. A quantitative discussion of these effects are beyond the scope of the present work, but will be the subject of future investigations.

We have also computed the electrical density current J in the presence of the time dependent interaction studied throughout this work. Using a standard definition of J [17] we obtained the following remarkably simple expression:

$$J(x,t) = C\lambda\,\Theta(t)\,\Big(\Theta(-x-\frac{a}{2})\,\sin[\Omega(t-x-\frac{a}{2})] - \Theta(-x+\frac{a}{2})\,\sin[\Omega(t-x+\frac{a}{2})] + \\ + \Theta(x+\frac{a}{2})\,\sin[\Omega(t+x+\frac{a}{2})] - \Theta(x-\frac{a}{2})\,\sin[\Omega(t+x-\frac{a}{2})]\Big), \quad (14)$$

where C is a renormalization constant. Let us stress that this is an exact result (in the absence of backscattering no perturbative expansion is required). We see that the current originated by the impurity (note that no external voltage has been applied) is a simple superposition of harmonic contributions and therefore its temporal average vanishes. If one introduces a bias V one obtains a (time-independent) term linear in V plus a photocurrent contribution similar to the above expression but with additional factors that depend on the right and left chemical potentials. The temporal average of this photocurrent is still zero. We then conclude that the dc conductance of the system is not affected by the perturbation. Again, this result is not changed by backscattering from the impurity, at least up to first order in the couplings.

Turning back to the TED, we have extended the previous analysis to the case of a Luttinger liquid. Performing the same kind of manipulations as in the non interacting case, we found an expression similar to the one corresponding to the $\gamma = 0$ case (eq. (13)). Since it is another lengthy expression we shall not write it down here. The only relevant difference is the appearance of decaying factors of the form $\exp(-\Lambda/v|\omega \pm n\Omega/2|)$ (Λ is

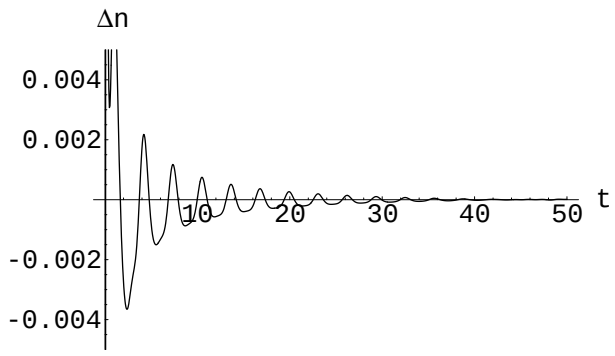

Figure 4: : $\Delta n(t)$ in the low energy regime, for $\gamma = 1/2$, $\Omega = 1$, $\omega = 0.01$.

an ultraviolet cutoff and v the renormalized velocity), characteristic of the smoothing of the energy density as consequence of the electron-electron interaction. One can also verify that $n_{\gamma\neq 0} < n_{\gamma=0}$. Another interesting point we want to mention concerns the comparison between the areas under the curves of n and n_0 (TED's with and without the time-dependent impurity) as functions of t for both the interacting and non-interacting electrons. In the first case the presence of the impurity leads to a smaller value for the total area, whereas a non vanishing forward scattering ($\gamma \neq 0$) produces the opposite effect. This occurs with and without a finite-time non adiabatic switching.

In Figure 4 we show $\Delta n(t)$ for $\gamma = 1/2$, $\Lambda = 0.1$ and $v = 1$. We kept the same values used in Figure 3 for the other parameters. One observes that $\Delta n(t)$ takes much lower values due to the fact that, in the presence of electron-electron forward-scattering, the TED's are smaller as consequence of the decaying factors mentioned above. On the other hand, comparing figures 3 and 4, it becomes apparent that the relaxation time is drastically diminished by the interaction. Then we conclude that for a Luttinger liquid with sufficiently large γ the effect of the finite-time switching can be safely neglected.

In summary, we have considered the effect of a harmonic time-dependent perturbation on the TED of a 1D fermionic system. We put special emphasis on the case in which the perturbation is switched on suddenly at a finite time, and compared our results with the ones obtained when the dynamic interaction is present at all times. The low-energy transient process by which the TED corresponding to finite-time switching evolves to the long-time (harmonic) TED is most clearly characterized by the difference $\Delta n(t)$. By carefully analyzing this function for different energies ω we were able to determine that the relaxation time t_R associated to the TED evolution, for free fermions ($\gamma = 0$), is equal to $1/(2\omega)$. We also established that for Luttinger liquids ($\gamma \neq 0$) t_R is significantly shortened. This suggests that, in order to observe some consequence of the transient

process, it is convenient to consider Luttinger systems with values of γ as small as possible. Concerning the transport properties, we showed that the electrical current originated by the time-dependent perturbation is a simple superposition of harmonic contributions. This means that if one introduces an external voltage V (through appropriate chemical potentials) the dc conductance will remain unchanged by the dynamic perturbation.

Acknowledgements

This work was partially supported by Universidad Nacional de La Plata and Consejo Nacional de Investigaciones Científicas y Técnicas, CONICET (Argentina).

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